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ABSTRACT

This paper briefly reviews some of the literature on the use of visual imagery in the conceptualization and solution of mathematical problems, especially its use in the learning of mathematics by children, and notes that a mutually facilitative effect occurs between mathematics and visual aptitude. A bibliography, a short list of resource materials, and samples of relevant curriculum materials are provided. (FM)

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VISUAL IMAGERY MEDIATION FOR LEARNING MATHEMATICAL CONCEPTS AND PROBLEM SOLVING

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ABSTRACT

The use of visual imagery has long been recognized as a valuable tool in the conceptualization and solution of mathematical problems. The primacy of visual mediation in children requires the use of visual representations to facilitate the understanding of abstract mathematical concepts; a variety of materials may be used for this purpose. Higher visual aptitude seems to be correlated with higher mathematical achievement, and students can be encouraged to succeed in learning mathematics through visual literacy programs.

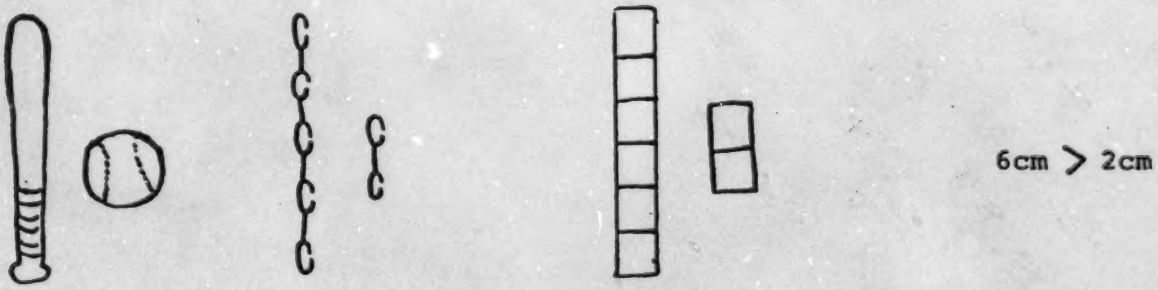
The use of visual imagery in mathematics and mathematical problem solving is not a new concept. The history of science is replete with instances of the employment of visual imagery in the solution of problems. Pythagoras (c.582-c.497 B.C.) demonstrated what is often called the most important theorem in mathematics. It is thought that the Pythagorean theorem ($a^2 + b^2 = c^2$) was first represented in visual form (Brownowski, 1973). Though the concept of the five regular solids (the tetrahedron, hexahedron, octahedron, dodecahedron, and icosahedron) was first developed by the Pythagoreans, Plato (c.427-c.347 B.C.) used these regular solids to represent each of the four elements, fire, earth, air, and water and the universe as a whole (Lloyd, 1970). Centuries later Johannes Kepler (1571-1630) devised a planetary system based on the five, regular, Platonic solids (Kearney, 1971). Isaac Newton's (1642-1727) own story relates how the sight of a falling apple led him to wonder if the same force that pulled the apple to the earth also held the moon in its orbit (McKie and de Beer, 1952). Newton later mathematically described the universal principle of gravitation ($F \propto \frac{m_1 m_2}{d^2}$).

Leonhard Euler (1707-1783), a Swiss mathematician, developed a method to define traceable graphs or Euler paths using a mathematical theory of graphs. Euler heard of the Königsberg bridges problem which asked if it was possible to travel through the town of Königsberg and cross the seven bridges without

recrossing any one of them. Jacobs (1970) has described the analysis of the problem:

> "...Euler began by redrawing the map so that the four areas of land were represented by four points and the seven bridges by seven lines joining the points." (p.458)

The structure of mathematics learning begins on a concrete level and develops to increasingly abstract levels. A mathematical concept may be taught most effectively by proceeding from the use of a concrete object to a physical representation of that object, to a pictoral representation and finally to a symbolic representation (Developing Mathematical Processes, Rand McNally & Company, 1974).



Comparing Length

It has been observed that young children do not approach a situation with the same priority of language as adults. Children conceptualize many situations through visual and tactile mediation rather than through verbal language. Children's visual and tactile competencies, at early ages, are more highly developed than their verbal language competencies (Donaldson, 1978).

As sensory experience enhances language development so, too, it enhances the development of observation, perception, and non-verbal relations which form the bases of skills in mathematics and mathematical problem solving. If learning occurs through sensory experience then visual imagery can provide a way to experience mathematics, and a way to use mathematics the way it is used, frequently, by those whose adult, specialized jobs involve using it.

Visual imagery mediation in mathematics involves the use of a variety of materials that encourage attainment of problem solving skills and enhance acquisition of concepts. The following curriculum materials are among those which have been effectively used to teach elementary mathematics.

Tangrams

Hexaflexagons

Strategy Games

Polyominoes

Motion Geometry and Tesselations

Graph Theory

Topological Puzzles

Sample materials are included in Appendix A, and a short listing of resources is included in Appendix B.

As a strategy for problem solving, Polya (1957) suggests drawing a picture or diagram to represent the data of the problem. DeBono (1970) emphasizes the value of such a visual approach.

> "...the advantage of drawing is that there is more commitment than with verbal explanation. Words can be general but a line has a definite place." (p. 282-283)

Davidson (1976) cites examples which demonstrate the usefulness of analogy and metaphor in learning. Using results from research on remembering, he asserts that metaphorical and analogical devices are schemata which can provide a basis for comprehension of abstract or complex information.

Among the problem solving strategies presented by Polya (1957) are the following suggestions: Looking at the problem and thinking of a similar problem that may have been solved previously, and viewing the problem on a smaller scale. Other problem solving strategies taught by math educators include drawing pictures or diagrams using the data, and dramatizing a problem situation.

Traditional instruction of mathematics has most often involved computation practice, rule application, and algorithm search; these analytical, sequential competencies alone do not produce problem solving skills. Recent studies in brain research have indicated that one brain hemisphere carries out analytical, sequential, logical processes, while the other hemisphere carries out holistic, simultaneous, abstract processes. Effective problem solving employs analytical and holistic processing, the use of both brain hemispheres (Wheatley, 1977).

The curriculum materials listed above are activities which encourage holistic and abstract thinking and imagery.

It has been found that students with high visual aptitude demonstrate higher achievement in mathematics than students with lower visual aptitude (Bruning, 1974). Children who have had early experiences with a variety of visual activities such as puzzles and games of a spatial nature appear to achieve greater success in school than those who lack such experiences (Thompson, 1973). However, children who lack a wide range of early visual experiences can be helped to attain success in school through participation in visual literacy programs (Fransecky and Ferguson, 1973).

There seems to be a mutually facilitative effect occurring between mathematics and visual aptitude. Many of the principles of mathematics and the analytical skills required in mathematics, developed through the visual mediation of mathematical problem solving, will also contribute reciprocally to a general visual literacy.

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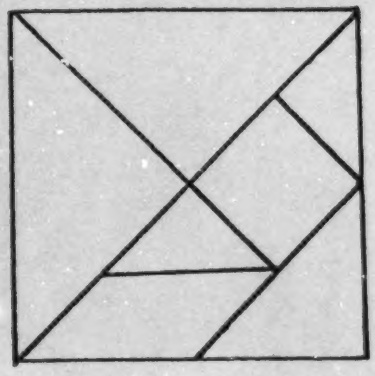
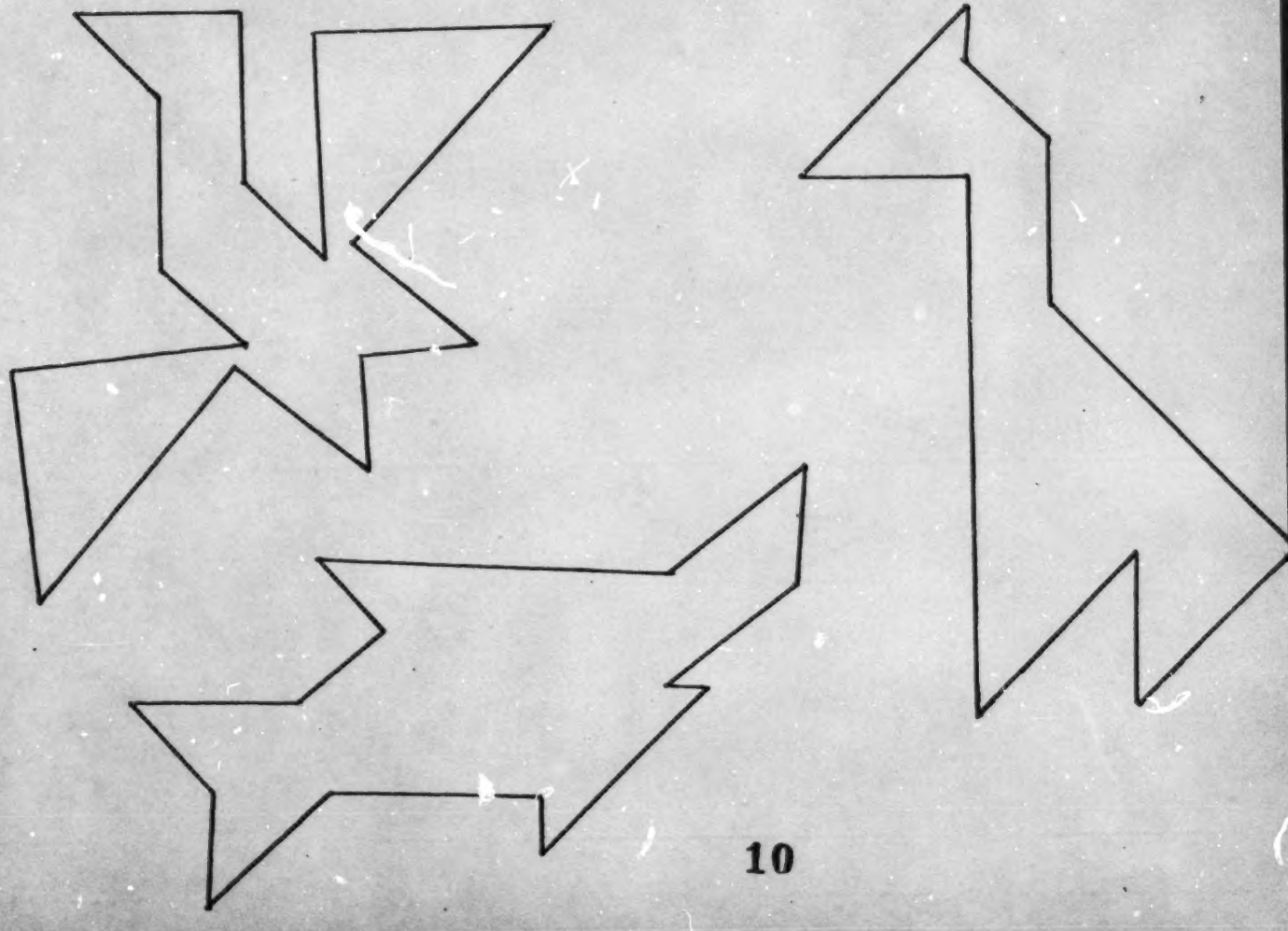
TANGRAMS



Seven geometric shapes make up this ancient Chinese puzzle. The shapes that make the square above can be placed together to form over 300 designs.

Cut out the pieces.

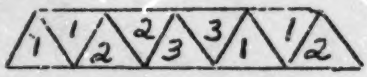
Can you use the seven tangrams to make these figures?



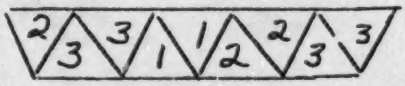
DIRECTIONS FOR MAKING A HEXAFLEXAGON

1. Carefully cut out the <u>entire</u> configuration below. (DO NOT cut out individual triangles!).

2. Number the triangles so they look like this.

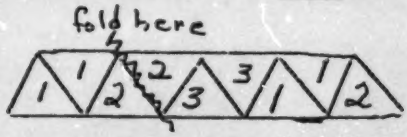


3. Turn the strip of triangles over and number as shown below.

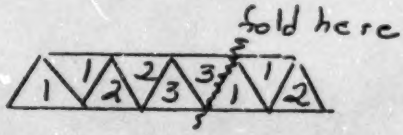


4. Carefully fold along each of the lines forming the triangles in the strip.

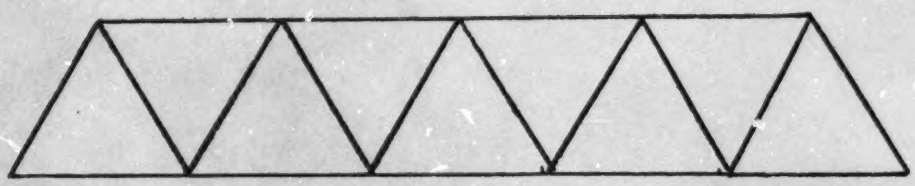
5. Turn the strip over so it looks like the figure in #2 above. Carefully fold along the line shown below. (Fold the 2 <u>over</u> the 2).

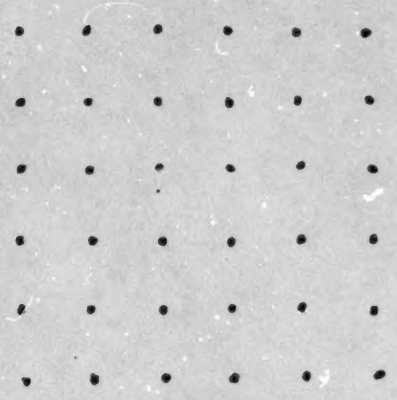


6. Next, make a second fold along the line as shown below. (Fold the 1 <u>back Under</u> the 3).



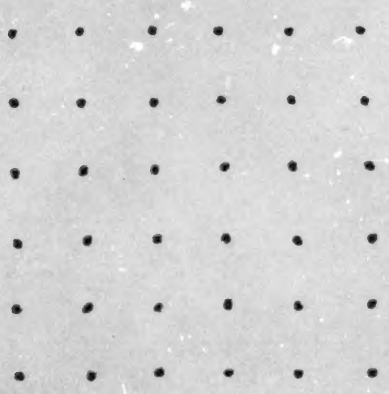
7. Finally, tuck the 2 <u>under</u> the 3. If you have done the folding correctly, you should have a flexagon with all 1's on one side and all 3's on the other. You are now ready to "play" with your flexagon.





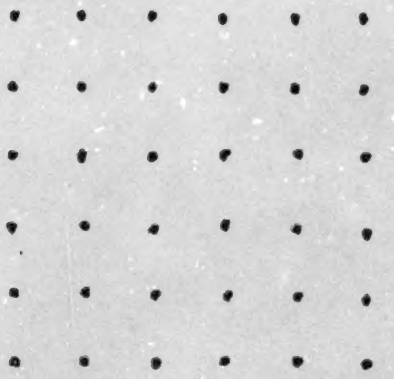
Creep

1. draw a line any length across, up, down, or diagonally.
2. do not cross another line or close the shape or turn a corner on one turn.
3. draw your line from either end of what is already there. (only straight lines)
4. winner is whoever draws last possible line.



Links

1. link any 2 dots that are next to each other, side-by-side (above or below).
2. a dot may be linked to only one other dot.
3. winner is whoever makes the last link.



Square Wins

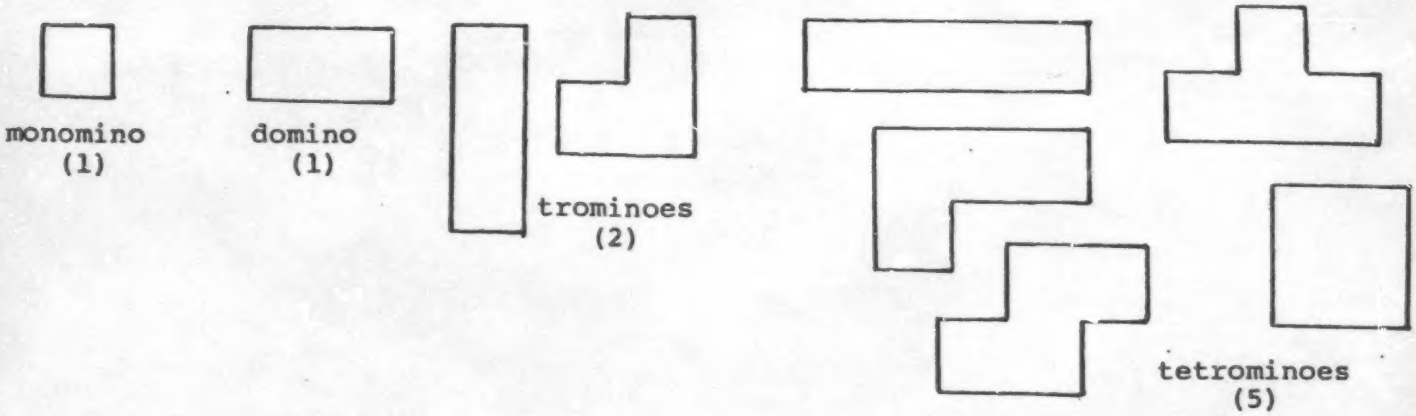
1. mark any dot on your turn.
2. one player uses X, the other uses O.
3. whoever marks 4 dots that would make a square, if connected, wins.

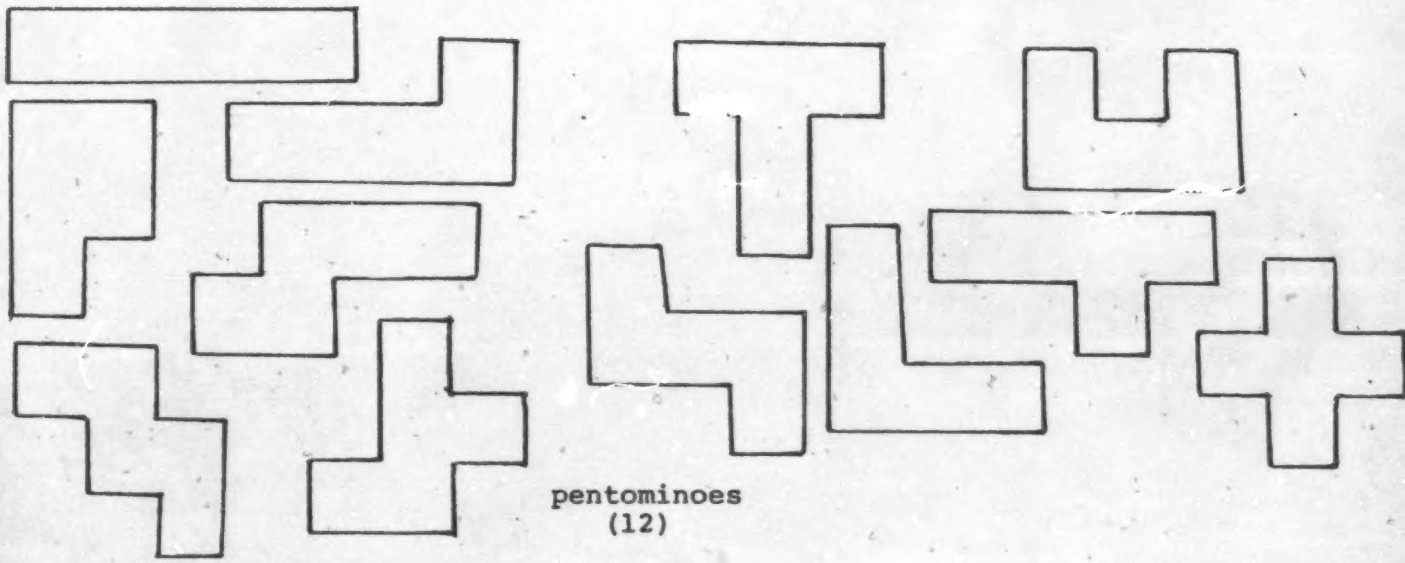
Sprouts

1. start with 2 points; take turns drawing lines.
2. a line must start and end at a point.
3. can use curved lines and loop
4. after drawing a line, make a new point somewhere on it.
5. no line can cross itself or another line, or pass through any point.
6. no point can have more than 3 lines ending there.
7. winner is the last person able to play.

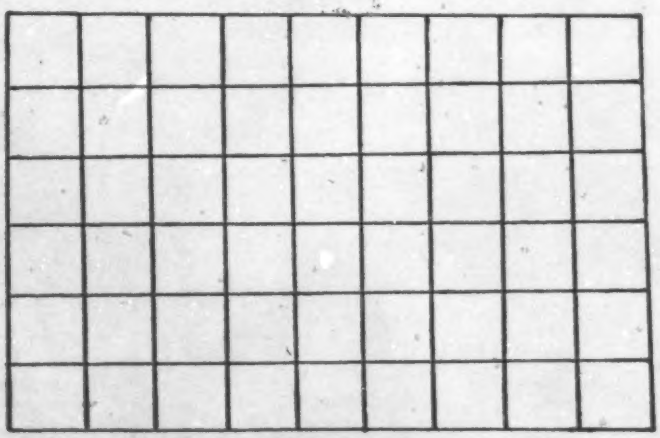
POLYOMINOES

Polyominoes are defined as a set of squares joined along their edges. The first five sets of polyominoes will be shown here.





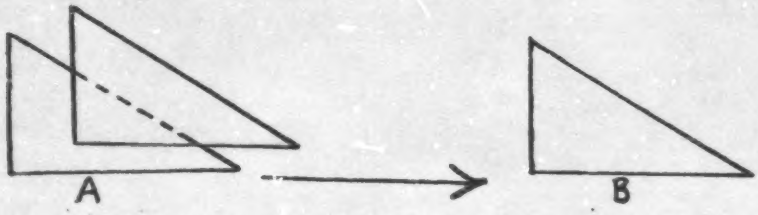
Fit all twelve pentominoes on a 6 x 10 rectangular grid. There are more than three thousand solutions to this puzzle.



MOTION GEOMETRY

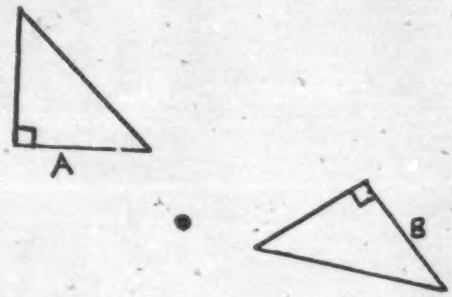
Slides

A sliding motion can be demonstrated below.
Place a triangle cutout on triangle A. This cutout can be made to coincide with triangle B by sliding it along the paper without a turn or twist. Triangle B is the slide image of triangle A.



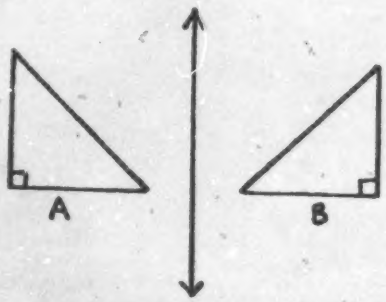
Turns

If triangle A is traced on a sheet of tracing paper and rotated 90° clockwise about the point then it will coincide with triangle B. Triangle B is the turn image of triangle A.

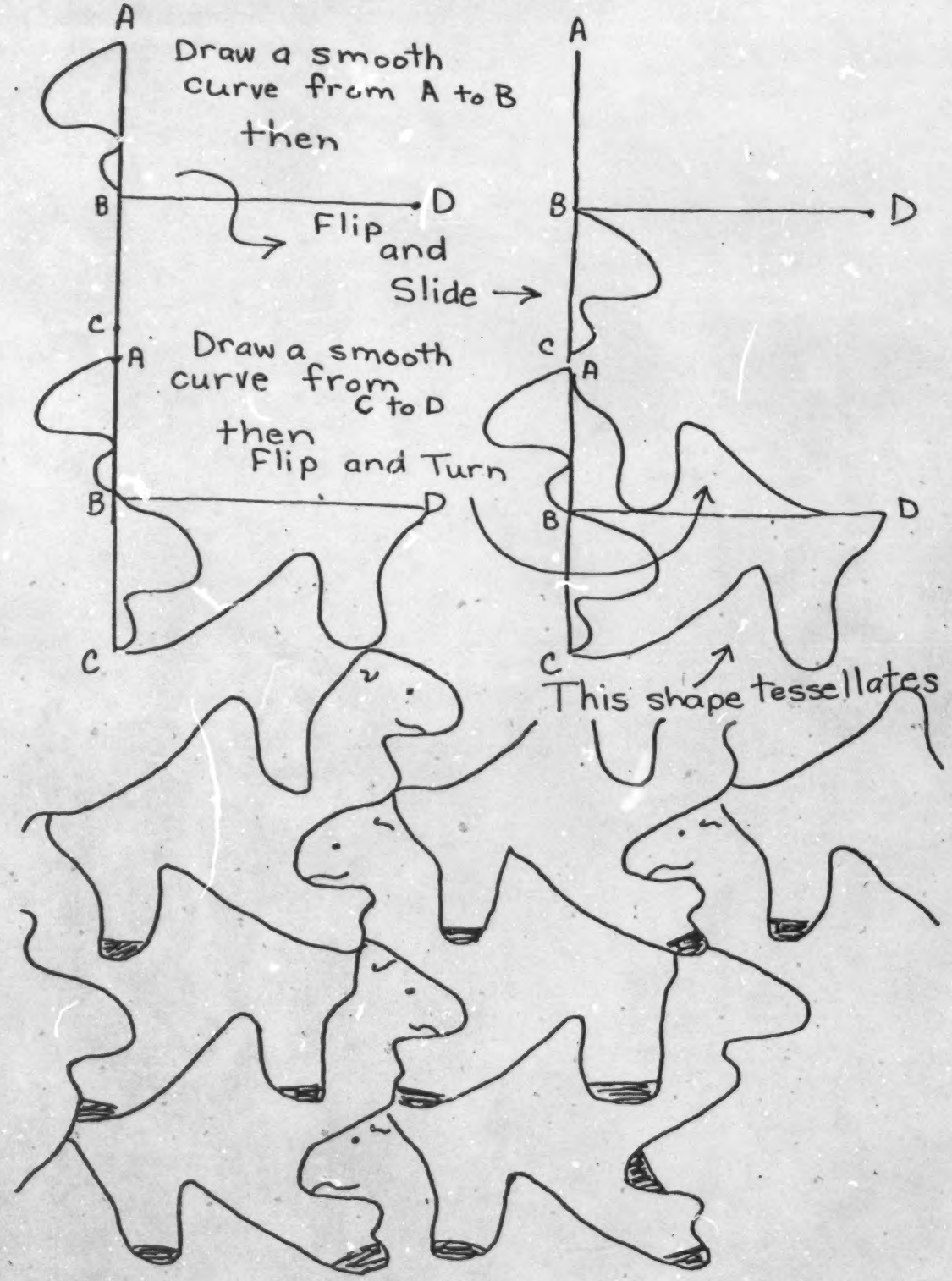


Flips

If a triangle cutout is placed to coincide with triangle A, it would need to be "flipped" in order to make it coincide with triangle B. Each flip involves a flip line, therefore a certain flip is indicated by the flip line.



Tesselating



GRAPH THEORY

Euler's Formula

Find a relationship between the number of vertices, paths, and regions of these networks.

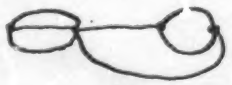
V vertices are the points in the network
P paths are lines or curves that connect points
R regions are the areas into which the network divides the plane

example

1

2

3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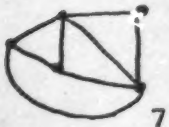
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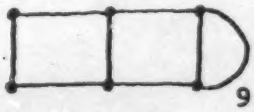
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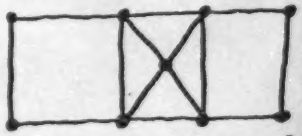
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Fill in the charts using the networks above.

Network	V	P	R	Even vertices	Odd vertices	Traceable?
Example	4	6	4	0	4	no
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

If the number of lines at a point is even then it is an even vertex, otherwise it is an odd vertex.

Traceable Network

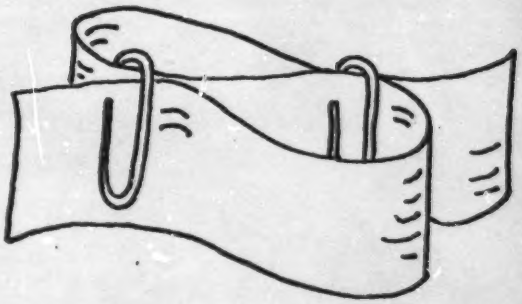
Begin at one point, trace each line one and only one time without lifting your pencil from the paper.

Can you find the patterns?

TOPOLOGICAL PUZZLES

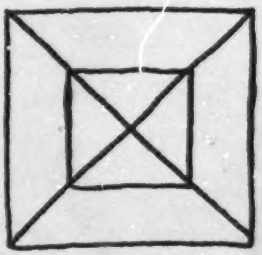
Embracing Paperclips

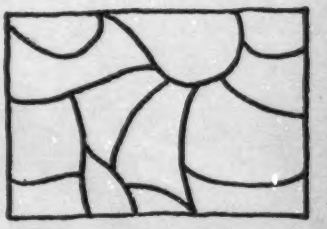
-use a strip of paper or a dollar bill
-curve it and clip paper clips to it like this
-hold the ends of the paper and pull hard
-catch the paper clips

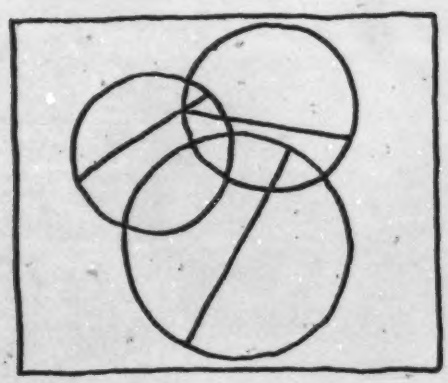


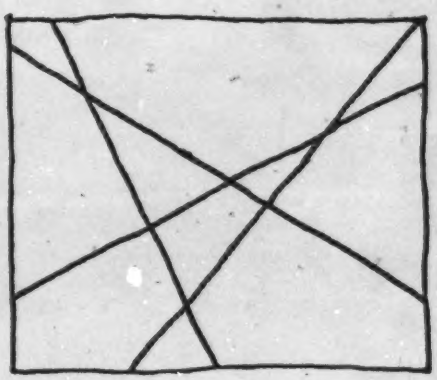
The Four Color Problem

-color these maps using the fewest colors possible (4 or less)
-no two regions with a common boundary have the same color
-regions that meet only at 1 point can be the same color









The Escape Puzzle

Loosely tie two heavy strings to the arms of 2 people. Interlock the strings. Without untying or cutting the strings, the victims must free themselves from each other.

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